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Reentrant superconductivity in the strong coupling limit

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The usual approaches to the reentrant superconductivity are based on the mean field approximation. Here, we investigate this effect carrying out exact diagonalization of finite systems. We have found a non-monotonic field dependence of the pair susceptibility both in the weak and strong coupling limit. This result strongly supports the possible onset of the reentrant superconductivity in the presence of strong magnetic field.

1 Introduction

It has commonly been accepted that superconductivity is suppressed by an external magnetic field. This basic property of the superconducting ground state is related to the breaking of global U(1) symmetry. The superconducting order parameter $\psi(\boldsymbol{r})$, introduced by the Landau–Ginzburg theory, becomes frustrated in the presence of magnetic field, what results in the emergence of supercurrents. Although orbital frustration is an inherent property of the superconducting state, magnetic field does not always lead to a suppression of superconductivity. At the end of 1980's it was suggested for multivalley semiconductors and semimetals, examined by Cohen [1], that sufficiently strong external magnetic field may drive the electron–phonon system into a novel triplet superconducting state [2]. Despite the results presented in the classic paper by Abrikosov [3], based on the negligence of the Landau-level structure, it was shown that including this structure gives rise to the reentrance of superconductivity in high magnetic fields [4]. This entirely new state of type-II superconductors reveals the breakdown of the London relation between the supercurrent and the vector potential. Moreover, it has been proved that superconducting transition temperature of a BCS superconductor remains finite for arbitrarily strong field [4]. In most cases, a finite temperature or impurity scattering totally erases any signature of Landau-level structure, since the number of occupied levels is huge and the distance between them is small. However, in low-carrier-density semimetals or semiconductors the full Landau-level structure needs to be taken into account.

A thorough theoretical discussion of the superconductivity in high magnetic fields is presented in Ref. [5], where the authors point out the conditions which need to be fulfilled for the experimental implementation of the reentrant state. The relation between superconductivity and strong magnetic field has also been investigated for the electron gas on a square lattice. Within the mean-field approximation it has been shown that the reentrant superconductivity may occur also in the presence of a strong periodic lattice potential [6, 7].

On one hand, the usually applied mean field approximation limitates the allowed values of pairing potential. On the other hand, the Landau level structure occurs only in quasi two-dimensional systems. An additional movement of electrons along the external magnetic field would be responsible for broadening of the Landau levels, which eventually may overlap. However, it is well known that in low dimen-

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sional systems, mean-field approaches may be invalid even for moderate many-body interactions. Therefore, we tackle the problem of the reentrant superconductivity carrying out exact diagonalization of finite clusters. We investigate, whether the reentrant behaviour occurs beyond the mean field approximation, especially, in the strong coupling limit.

2 Model and the details of calculations

Our starting point is a 2D system described by the attractive Hubbard model in the presence of a perpendicular magnetic field [8]:

$$H = -\sum_{\langle i,j\rangle\sigma} t_{ij}(\boldsymbol{A})\, c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow} - \mu \sum_{i\sigma} n_{i\sigma} \,. \tag{1}$$

Here, $n_{i\sigma} = c^{\dagger}_{i\sigma} c_{i\sigma}$ and $t_{ij}(\boldsymbol{A})$, hopping matrix elements, depend on the choice of vector potential. The magnetic field is coupled to the system via Peierls substitution [9]. The Landau gauge was chosen, $\boldsymbol{A} = B(0, x, 0)$. In the case of finite system calculations, periodic boundary conditions can be applied only for specific values of the magnetic field, which depend on the cluster size. Therefore, we have used the fixed boundary conditions.

In the following we carry out exact diagonalization of finite clusters described by the above Hamiltonian. In such small systems superconducting phase transition does not occur. Therefore, the investigations of the reentrance effect were based on the analysis of the Cooper pair susceptibility. In the case of the on-site attraction one usually calculates the pair susceptibility in the following form [10]

$$\chi_{\text{sup}} = \frac{1}{N} \sum_{ij} \left(\langle \Delta_i \Delta^{\dagger}_j \rangle - \langle c_{i\uparrow} c^{\dagger}_{j\uparrow} \rangle \langle c_{i\downarrow} c^{\dagger}_{j\downarrow} \rangle \right), \tag{2}$$

where the Cooper pair creation operator is given by $\Delta^{\dagger}_i = c^{\dagger}_{i\downarrow} c^{\dagger}_{i\uparrow}$ and N stands for the number of sites of the lattice. The increase of this quantity indicates that the pairing correlations are enhanced. However, this form cannot be used in the presence of the external magnetic field imposed on the lattice. In this situation we need a gauge invariant quantity. Therefore, we have investigated eigenvalues of the following Hermitian matrix

$$\chi_{ij} = \langle \Delta_i \Delta^{\dagger}_j \rangle - \langle c_{i\uparrow} c^{\dagger}_{j\uparrow} \rangle \langle c_{i\downarrow} c^{\dagger}_{j\downarrow} \rangle \,. \tag{3}$$

As it was argued in Ref. [11], one can use the maximal eigenvalue of the above matrix $\lambda_{\max}$ as a measure of the tendency toward the formation of the paired state. The averages in the matrix elements χ_{ij} are calculated with the exact eigenstates. The influence of temperature has been investigated within the canonical ensemble.

In the mean-field analysis that reentrant effect is directly related to the density of states (DOS). In order to calculate DOS, we used the spectral functions [12] for adding an electron of spin σ, momentum $\boldsymbol{k}$ and energy ω to the ground state

$$A^{(+)}_{\sigma}(\boldsymbol{k}, \omega) = \frac{1}{N} \sum_{n} |\langle \psi^{N_e+1}_n | c^{\dagger}_{\boldsymbol{k}\sigma} | \psi^{N_e}_0 \rangle|^2 \, \delta\left[\omega - (E^{N_e+1}_n - E^{N_e}_0)\right] \tag{4}$$

as well as for removing an electron from the ground state

$$A^{(-)}_{\sigma}(\boldsymbol{k}, \omega) = \frac{1}{N} \sum_{n} |\langle \psi^{N_e-1}_n | c_{\boldsymbol{k}\sigma} | \psi^{N_e}_0 \rangle|^2 \, \delta\left[\omega + (E^{N_e-1}_n - E^{N_e}_0)\right], \tag{5}$$

where $E^{N_e}_n$ and $\psi^{N_e}_n$ are the eigenenergy and eigenstates, respectively, of the system of N_e electrons. We have obtained all the eigenenergies and eigenstates by exact diagonalization of the Hamiltonian (1). The DOS of the system consists of two parts

$$\rho(\omega)^{\pm}_{\sigma} = \sum_{k} A^{\pm}_{\sigma}(\boldsymbol{k}, \omega) \,, \tag{6}$$

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corresponding to the spectral functions. When calculating the DOS at finite temperature, we have replaced the ground state $|\psi_0^{N_e}\rangle$ in Eqs. (4) and (5) with a summation over all eigenstates $|\psi_m^{N_e}\rangle$ with the weight $(1/Z)\exp(-E_m/k_B T)$.

3 Numerical results

The Cooper pair susceptibility calculated in the presence of the external magnetic field is presented in Figs. 1 and 2. The values of the λ_{max} were investigated for various cluster sizes, occupation numbers and the pairing potential U. One can see that λ_{max} strongly depends on the magnetic flux, that is described by the parameter $\alpha = \Phi/\Phi_0$. Here, Φ denotes the magnetic flux per unit cell and $\Phi_0 = hc/e$ is the flux quantum.

As a consequence of the Peierls substitution the Hamiltonian (1) is symmetric with respect to the value $\alpha = 1/2$. For that reason the susceptibility was investigated in the range of the α parameter from 0 to 1/2. Obtained results indicate that the behaviour of the Cooper pair susceptibility vs. the magnetic flux is clearly nonmonotonic. One can see that in the low field regime, increase of α leads to the suppression of the superconductivity, since the Cooper pair susceptibility decreases sharply. In the intermediate range of α the susceptibility exhibits plateau-like behaviour with the lowest values. However, with further increase of α the Cooper pair susceptibility increases as well, what indicates on the strong enhancement of the superconducting correlations. This enhancement is especially pronounced for larger clusters presented in Fig. 1. In this case λ_{max} calculated for $\alpha \simeq 0.5$ can even exceed the values obtained in the absence of magnetic field. One can see that enhancement of temperature hardly influences the field dependence of the susceptibility, provided $kT < 0.3t$. Higher temperature causes a washout of the reentrance effect. Qualitatively, our results do not depend on the cluster size and the occupation number (compare Figs. 1 and 2). Therefore, these findings can be interpreted as the evidence for the reentrant behaviour in the attractive Hubbard model.

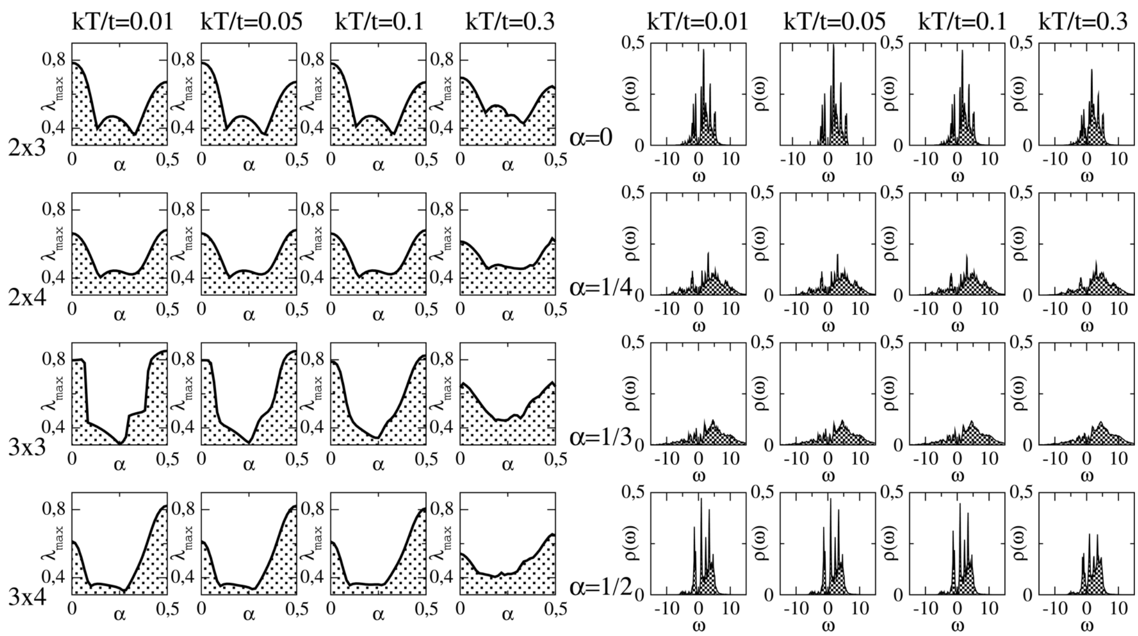

Fig. 1 Largest eigenvalue of the susceptibility matrix λ_{max} in the presence of magnetic field α for various cluster sizes and temperatures (left panel). The cluster sizes are explicitly depicted in the figure. The DOS of the 3×3 cluster in the presence of magnetic field α for various temperatures (right panel). Results have been obtained for the system consisting of 2 spin up and 2 spin down electrons with $U/t = -4$.

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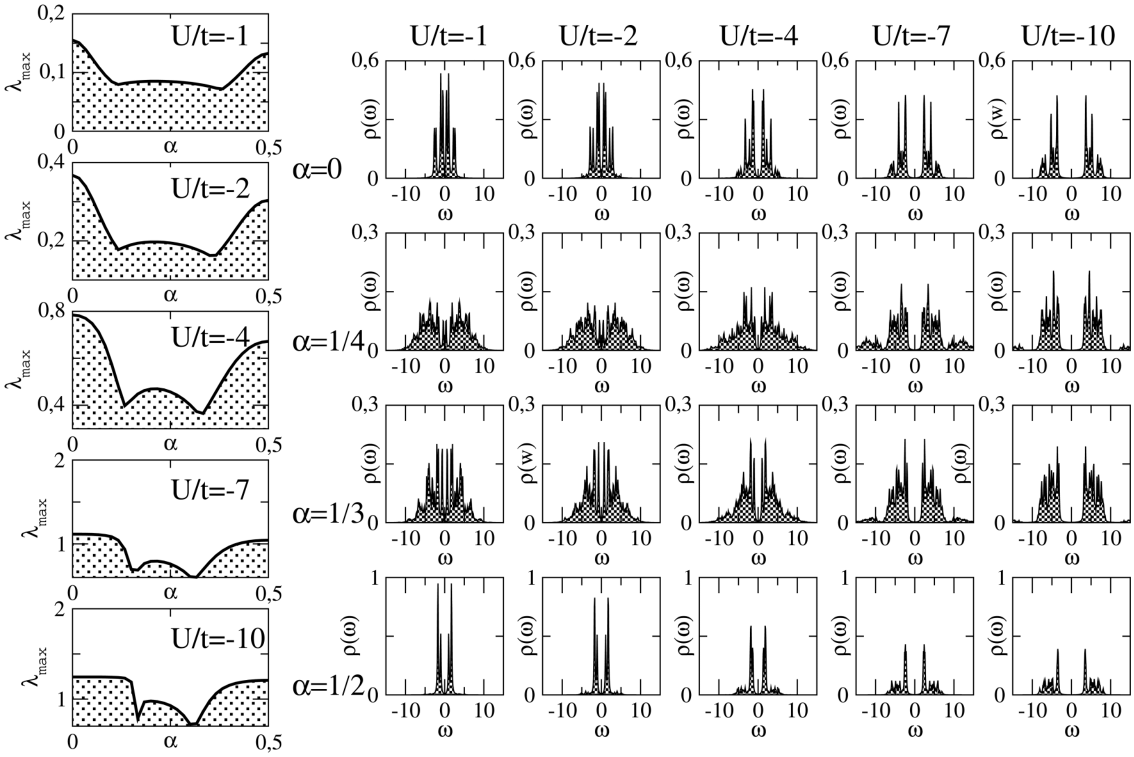

Fig. 2 Largest eigenvalue of the susceptibility matrix λ_{max} in the presence of magnetic field α (left panel) and the DOS $\rho(\omega)$ (right panel). The results have been obtained for the 2×3 cluster for half filling and various U. The temperature has been assumed at $kT/t = 0.01$.

As expected, the increase of U is responsible for an enhancement of the pair susceptibility and an increase of the energy gap at the Fermi level. When U increases the one-particle excitations are replaced by the collective ones with a finite live-time. Instead of the δ-like peaks in the density of states we obtained much larger number of wider peaks, which eventually may overlap. The relation between the reentrant behaviour and DOS is not as clear as in the case of free electron gas. However one can note that for $\alpha = 0$ and $\alpha = 1/2$ the maximal values of DOS are larger than for the intermediate range of α. The peaks stick out from the background. On the other hand, the DOS spectra for the intermediate values of α are more complicated and consist of much larger number of broader peaks. Therefore, also the magnitude of DOS is lower, what probably leads to the decrease of the susceptibility in the intermediate field regime.

It is worth to emphasize that the reentrant behavior takes place for a wide range of the pairing potential. Therefore, one may expect that this effect should occur also in the strong coupling limit, that can not be investigated within the mean-field approaches.

In the present approach we have neglected role of the Zeeman splitting, that changes the concen tration of electrons with opposite spins. However, the considered clusters are too small to provide a reliable description of this effect. Additionally, we have carried out the numerical calculations in the canonical ensemble assuming the number of spin up and spin down electrons. In the BCS limit this pair breaking mechanism alone may destroy superconductivity, when the magnetic field exceeds the Clogston–Chandrasekhar limit. However, for a sufficiently strong pairing potential superconductivity may survive also in the reentrant regime, as demonstrated in Ref. [7]. Therefore, the Zeeman splitting should reduce λ_{max}, however, its nonmonotonic field dependence should hold, at least in the strong coupling regime.

4 Summary

Contrary to the predictions of the Ginzburg-Landau theory, there is a regime in high magnetic fields in which a phase transition to a new superconducting state is possible. In order to investigate the reentrant effect one needs to include the Landau level structure. So far, a thorough examination of the reentrant superconductivity has been presented within the mean field approximation [5]. The nature of the orbital frustration is completely changed in this regime and the enhancement of superconductivity can take place through a density-of-state effect. In this paper we have investigated the reentrant superconductivity beyond the mean field approximation for 2D attractive Hubbard model. The pair correlation function has exactly been calculated for finite clusters described by the attractive Hubbard model. Although the exact diagonalization strongly limitates the cluster sizes, it gives a precise method of investigating low dimensional correlated systems. We have obtained a nonmonotonic field-dependence of pair susceptibility, what indicates that the reentrant effect is not an artefact of the mean field analysis. We have shown that this result remains valid also in the strong coupling regime, which is beyond the reach of the previous mean-field analysis. Since our qualitative results are independent of the cluster size, we expect they hold true also for much larger systems.

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