Reply to "Comment on 'BCS-type mean-field theory for the t-J model in the SU(2|1) superalgebra representation' "

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We show that the SU(2|1)-superalgebra path-integral representation for the partition function of the t-J model follows from the conventional slave-fermion path-integral representation under a change of variables that explicitly resolves the constraint of no double occupancy.

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In Ref. 1 we use the SU(2|1)-superalgebra path-integral representation for the partition function of the t-J model to derive, under some simple mean-field approximation (MFA), a phase diagram $T_c(\delta)$. Our basic point is that within the SU(2|1) theory we rigorously take into account a crucial for the t-J model requirement of no doubly occupied electron states. The basic statement in the Comment is, however, that our MFA violates the constraint and results in "an physical phase diagram." Moreover, the authors of the Comment claim that our approach is identical to that based on the slave-fermion hard-core boson (SFHCB) representation of the Hubbard operators.

In the present Reply we show that the constraint of no double occupancy is explicitly resolved in terms of the SU(2|1) path-integral representation used in Refs. 1 and 2. It then follows that the criticism voiced in the Comment is basically an artifact of the author's misunderstanding of this simple observation.

We start with the path-integral slave-fermion representation of the t-J partition function. Basic ingredients that enter the path-integral action are the classical symbols of the slave-fermion Hubbard operators. Let $X_{\lambda\lambda'}, \lambda=1,2,3$ be a 3×3 matrix of the Hubbard operator X. Consider a complex composite vector $\bar{d}=(\bar{a},\bar{b},\bar{f})$, where a and b stand for complex bosonic amplitudes, and f denoted a fermionic one. Then, the slave-fermion representation reads $X^{cl} = \Sigma_{\lambda\lambda'}\bar{d}_\lambda X_{\lambda\lambda'}d_{\lambda'}$, where

$$\sum_\lambda \bar{d}_\lambda d_\lambda = \bar{a}a+\bar{b}b+\bar{f}f=1. \qquad (1)$$

The last equation is just the classical counterpart of the constraint of no double occupancy. Let us now make a change of variables that explicitly resolves constraint (1):

$$a=\frac{1}{\sqrt{1+|z|^2+\bar{\xi}\xi}}, \quad b=\frac{z}{\sqrt{1+|z|^2+\bar{\xi}\xi}},$$

$$f=\frac{\xi}{\sqrt{1+|z|^2+\bar{\xi}\xi}}, \qquad (2)$$

where z is a complex number and ξ is an odd-valued Grassmann parameter. It can then be easily shown that under change of variables (2) the slave-fermion path-integral representation of the partition function reduces to the SU(2|1) one derived in Ref. 2 and employed in Ref. 1. Geometrically, the set (z,ξ) appears as local (projected) coordinates of a point on the supersphere defined by Eq. (1). For *any* given map $t\to(z_i(t),\xi_i(t))$ (we restore here the site dependence), including that of the MF type $t\to(z_i^{(0)},\xi_i(t))$ employed in Ref. 1, point $(z_i(t),\xi_i(t))$ always belongs to the supersphere. The local constraint of no double occupancy is therefore rigorously taken into account in Ref. 1; also see Ref. 4. Being explicitly resolved, this constraint neither can be subject to any factorization, nor can it be violated putting $z_i=z_i^{(0)}$. Of course, in the MFA we are unable to recover "the rigorous correlations for the fields at the lattice site," but this is basically a matter of dynamics: rigorous incorporating of the local constraint cannot by itself recover the exact correlators.[7]

Furthermore, it is quite natural that we obtain the odd-parity symmetry of the spinless fermion order parameter (OP) $\langle f_i f_j\rangle$: two spinless fermions cannot be placed on the very same lattice site. This symmetry, however, is not identical with a symmetry of the gap experimentally detected in cuprates. Equation (5) in Ref. 1 determines the spectrum of spinless fermions (holons), whereas a real electron within the resonating valence bond (RVB) scenario is a convolution of a holon and spinon. Accordingly, the electron OP $\Delta_{ij}^{(s,d)} = \langle f_i f_j\rangle\Lambda_{ij}^{(s,d)}$, where the spinon OP $\Lambda_{ij}^{(s,d)}$ describes the short-range antiferromagnetic spinon fluctuations.[5] It has correct transformation properties, $\Delta^{(s,d)}(\vec{k}) \propto \Sigma_{\vec{q}}\langle f_{\vec{q}} f_{-\vec{q}}\rangle\ \beta_{\vec{q}+\vec{k}}^{(s,d)} \propto \cos k_x \pm \cos k_y$, where $\beta_{\vec{k}}^{(s,d)}=\sin k_x \pm \sin k_y$. Within the adopted approximation $\Lambda_{ij}\neq0$,[7] and in the SC phase $\Delta_{ij}^{(s,d)}=0$ as soon as $\langle f_i f_j\rangle=0$. This enables us to determine $T_c(\delta)$ starting from Eqs. (6)–(8) in Ref. 1. Note that within our approach $\langle X_i^{+0}X_i^{-0}\rangle^{(s,d)} \propto \Sigma_{\vec{k}}\Delta^{(s,d)}(\vec{k})\equiv0$ for both the nn and nnn pairing contrary to the statement in the Comment. Note also that the maximum in $T_c(\delta)$ is shifted in Ref. 1 towards very small values of the hole concentration δ just because we have not taken into account the existing at a small doping long-range antifferomagnetic (AF) order.

Within the SFHCB representation[9] the spinon OP is given by Eq. (45). Within the MFA it is evaluated to be

$$\varphi_{ij}=\frac{z_i^{(0)}\overline{z_j^{(0)}}-z_j^{(0)}\overline{z_i^{(0)}}}{(1+|z_i^{(0)}|^2)(1+|z_j^{(0)}|^2)}\neq\Lambda_{ij}$$
$$=\frac{z_i^{(0)}-z_j^{(0)}}{\sqrt{(1+|z_i^{(0)}|^2)(1+|z_j^{(0)}|^2)}},$$

and despite their similar appearance Eqs. (51) and (52) in Ref. 9 and Eqs. (6) and (7) in Ref. 1, have quite a different spin content. This difference originates from the fact that within the SFHCB representation the necessary constraint is implemented at the apparent expense of having an additional unphysical state which, if not excluded, contributes to the partition function and correlators.[10] In summary, the SFHCB approximation and the SU(2|1) path-integral representation are not equivalent.

Finally, our critical remark[1] casts serious doubt on the reliability of one of the basic statements made in Ref. 3 that the s-wave order parameter is forbidden just due to the algebraic properties of the Hubbard operators. By definition, $X_i^{\sigma 0}X_i^{-\sigma 0}\equiv 0$. However, being averaged in Ref. 3 over a particular set of functions with the s-wave symmetry this identity surprisingly yields $\langle X_i^{\sigma 0}X_i^{-\sigma 0}\rangle\neq 0$. The only way to "reconcile" this with the above operator identity is just to claim that the s-wave pairing must be excluded. Here we can only add that there is a counterexample that explicitly supports our critical remark. The mean-field treatment of the AF Heisenberg model (written down in terms of the Hubbard operators) shows that the s- and d-wave order parameters are related by a gauge transformation.[8] As is known, this model reduces to the t-J interaction at half filling. As a "derivation" presented in Ref. 3 holds for any hole concentration, it obviously contradicts the exact result of Ref. 8.

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[1] E.A. Kochetov and M. Mierzejewski, Phys. Rev. B **61**, 1580 (2000).

[2] E.A. Kochetov and V.S. Yarunin, Phys. Rev. B **56**, 2703 (1997).

[3] N.M. Plakida *et al.*, Physica C **160**, 80 (1989); V.Yu. Yushankhai *et al.*, *ibid.* **174**, 401 (1991); N.M. Plakida, Philos. Mag. B **76**, 771 (1997); N.M. Plakida and V.S. Oudovenko, Phys. Rev. B **59**, 11 949 (1999).

[4] Note that within our approach all the on-site correlators that describe a simultaneous creation or annihilation of two spinless fermions on a given site are *identically* equal to zero, as they should, $\langle X_i^{+0}X_i^{-0}\rangle\sim\langle\xi_i^2\rangle\equiv 0$, etc. This is, however, not the case in Ref. 3.

[5] It can be shown that $\langle\vec{S}_i\vec{S}_j\rangle_{NN}=-|\Lambda_{ij}|^2/2+1/4$ in accordance with Ref. 6. Here $\vec{S}$ stand for the SU(2) spinon operators.

[6] Yi Zhou, V.N. Muthukumar, and Zheng-Yi Weng, Phys. Rev. B **67**, 064512 (2003).

[7] However, within the MFA in Ref. 1 we indeed can observe such a correlation: a probability for a spinon singlet to exist on the nn sites can be found to be $|\Lambda_{ij}|^2=1-\delta^2$.

[8] Ian Affleck, Z. Zou, T. Hsu, and P.W. Anderson, Phys. Rev. B **38**, 745 (1988).

[9] N.M. Plakida, Condens. Matter Phys. **5**, 707 (2002).

[10] In fact, within the SFHCB approximation the problem of the constraint of no double occupancy reasserts itself in the appearance of an unphysical degree of freedom. In particular, in Ref. 9 Hubbard operators are replaced in Eq. (37) by operators (29). However, the latter do not represent Hubbard operators, because Eq. (29) violates identity $X^{00}+\Sigma_\sigma X^{\sigma\sigma}=1$. They become Hubbard operators only upon projection onto the three dimensional particle-hole space; see, e.g., Ref. 2.